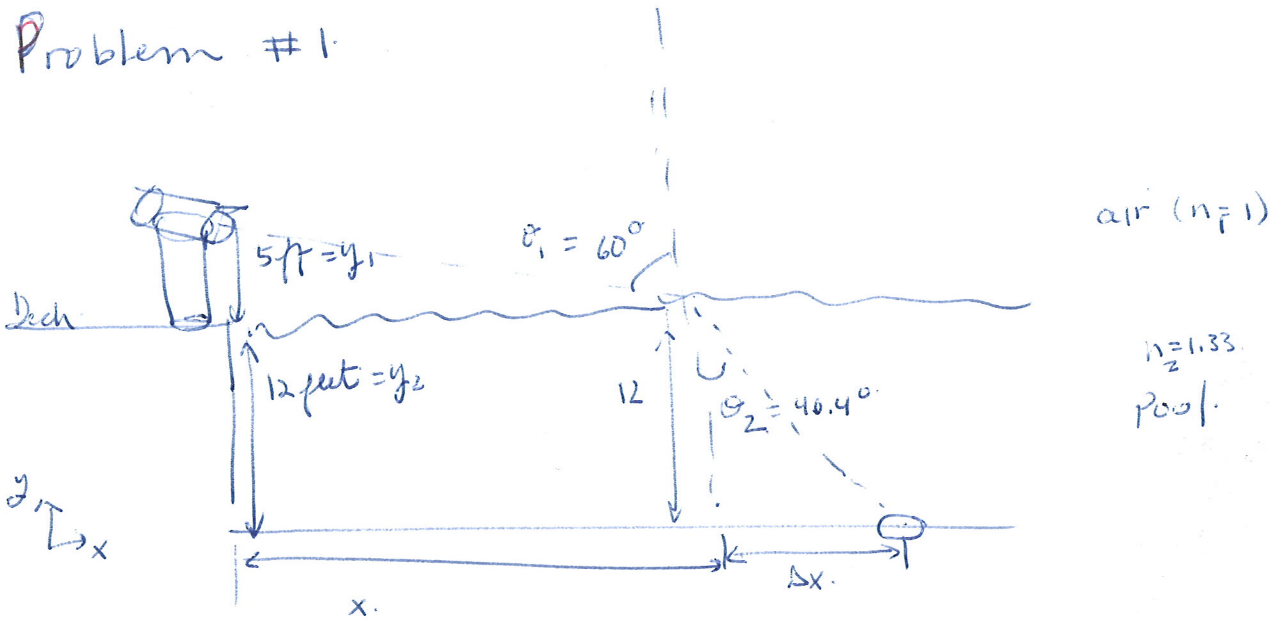
Problem #8 Solutions WPE Spring 2015

Problem #1.

0.5




Using snell's law, what is $\theta_2$.

1.0

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = \frac{1}{1.33}\sin 60^\circ$$

$$= 0.655$$

$$\theta_2 = \sin^{-1}(0.655)$$

$$= 40.4^\circ$$

Location $= x + \Delta x$.

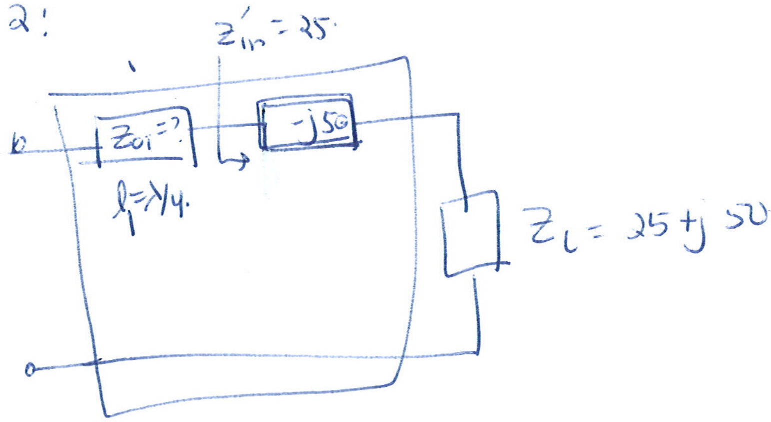
1.0

$x \Rightarrow \quad \tan\theta_1 = \dfrac{x}{y_1} \quad \Rightarrow \cancel{y} = y \tan\theta_1 = 5 \tan 60^\circ = 8.66$

$\Delta x \Rightarrow \quad \tan\theta_2 = \dfrac{\Delta x}{y_2} \quad \Rightarrow \Delta x = y_2 \tan\theta_2 = 12 \tan 40.6^\circ = 10.29.$

$$l = x + \Delta x = 8.66 + 10.29 = \boxed{18.94 \text{ ft} = l}$$

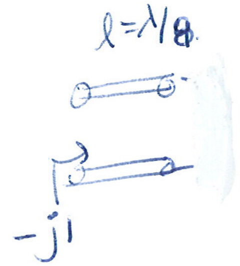
Problem 2:




0.5   $50\,\Omega$

0.5 A) $-j50 =$

$\bar{Z}_L = -j\frac{50}{50} = -j1 \Rightarrow$ capacitive $\Rightarrow$




0.5 B) $50\,\Omega$ section:

$Z_{01} = \sqrt{25 \cdot 50} = 35.35\,\Omega$

$l_1 = \lambda/4$

$= \frac{30}{4}$ cm

$\boxed{l_1 = 7.5 \text{ cm}}$

assume $\varepsilon_r = 1$

$\lambda = \frac{c}{f} = \frac{3 \cdot 10^8}{1 \times 10^9} = .3 \times 10^1$ m

$= 300$ m.

Problem 3.

$$Z_1 = -j25 = -j\frac{1}{\omega C}$$

$$C = \frac{1}{25\omega} = \frac{1}{25\cdot 2\pi f} = \frac{1}{50\pi \times 10^{9}} = \frac{10^{-9}}{50\pi}$$ desired.

$$C_x = \frac{\varepsilon_x A}{d} = \frac{\overset{4}{\cancel{\varepsilon_{rx1}}}\,\varepsilon_0\,(1)\times 10^{-6}}{d}$$ $A_x = w \times l = \frac{1\,mm^2}{5}$

$$C_y = \frac{\varepsilon_y A}{d} = \frac{\overset{1}{\cancel{\varepsilon_{ry}}}\,\varepsilon_0\,(1)\times 10^{-6}}{d}$$ $A_y = w \times l = \frac{1\,m^2}{5}$

$$C_T = 3C_x + 2C_y = \frac{12\,\varepsilon_0}{5d}\times 10^{-6} + \frac{2\,\varepsilon_0}{5d}\times 10^{-6}$$

$$= \left(\frac{14}{5}\varepsilon_0 \times 10^{-6}\right)\left(\frac{1}{d}\right)$$

$$d = \left(\frac{14}{\cancel{5}}\varepsilon_0 \times 10^{-6}\right)\times\frac{\overset{1}{\cancel{5}}0\pi}{10^{-9}}\,1/F$$

$$= 140\pi\,\varepsilon_0 \times \frac{\cancel{10^{-6}}}{\cancel{10^{-9}}}\,10^{3}$$

$$= (140\pi)\,8.85\times 10^{-12}\,\frac{(F/m)\cdot m^2}{F}\times 10^{3}$$

$$= 140\pi\cdot(8.85)\times 10^{-9}$$

$$\boxed{d = 3{,}89 \times 10^{-6}\ m}$$